Farfield Point Source

$\hat{\boldsymbol{l}}$ p-wave unit vector; $\hat{\boldsymbol{p}}$ SV unit vector; $\hat{\boldsymbol{\phi}}$ SH unit vector

$$\hat{\boldsymbol{\phi}} \times \hat{\boldsymbol{l}} = \hat{\boldsymbol{p}}; \quad \hat{\boldsymbol{p}} \times \hat{\boldsymbol{\phi}} = \hat{\boldsymbol{l}}; \quad \hat{\boldsymbol{l}} \times \hat{\boldsymbol{p}} = \hat{\boldsymbol{\phi}}$$

The simplest way to see what is happening is to imagine a single point source double-couple force system. Assume $\delta = 90°$, i.e., a vertical fault; assume $\lambda = 0°$, left-lateral fault; assume $\phi_s = 0°$.

$$\boldsymbol{u}^{P}(\boldsymbol{x},t) = \left[\sin^2 i_\xi \sin 2\phi / 4\pi\rho\ \alpha^3 r\right] \mu A \dot{\bar{s}}(t - r/\alpha)\hat{\boldsymbol{l}}$$

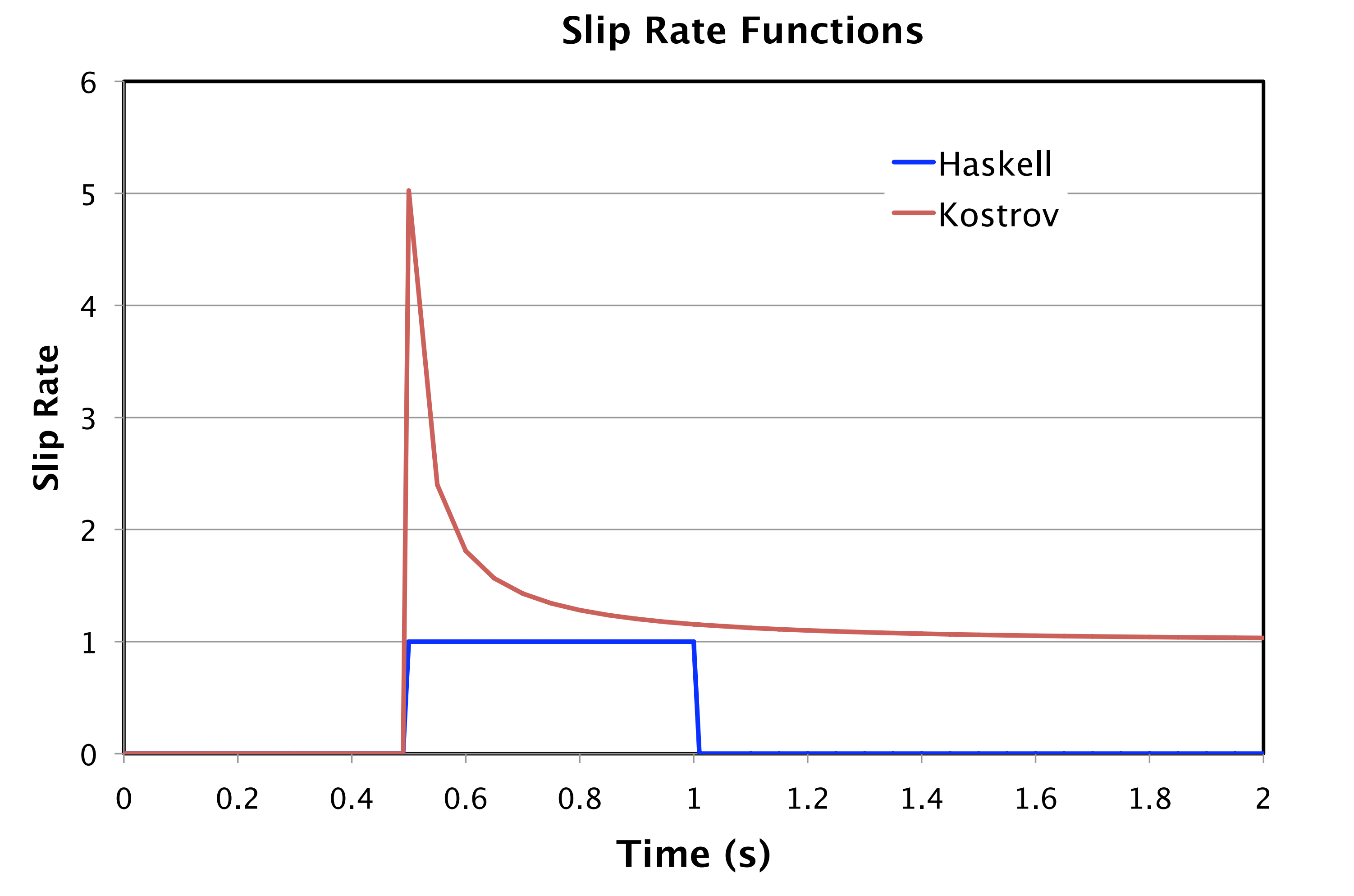
$$\boldsymbol{u}^{SV}(\boldsymbol{x},t) = \left[\sin 2\ i_\xi \sin 2\phi / 8\pi\rho\ \beta^3 r\right] \mu A \dot{\bar{s}}(t - r/\beta)\hat{\boldsymbol{p}}$$

$$\boldsymbol{u}^{SH}(\boldsymbol{x},t) = \left[\sin\ i_\xi \cos 2\phi / 4\pi\rho\ \beta^3 r\right] \mu A \dot{\bar{s}}(t - r/\beta)\hat{\boldsymbol{\phi}}$$

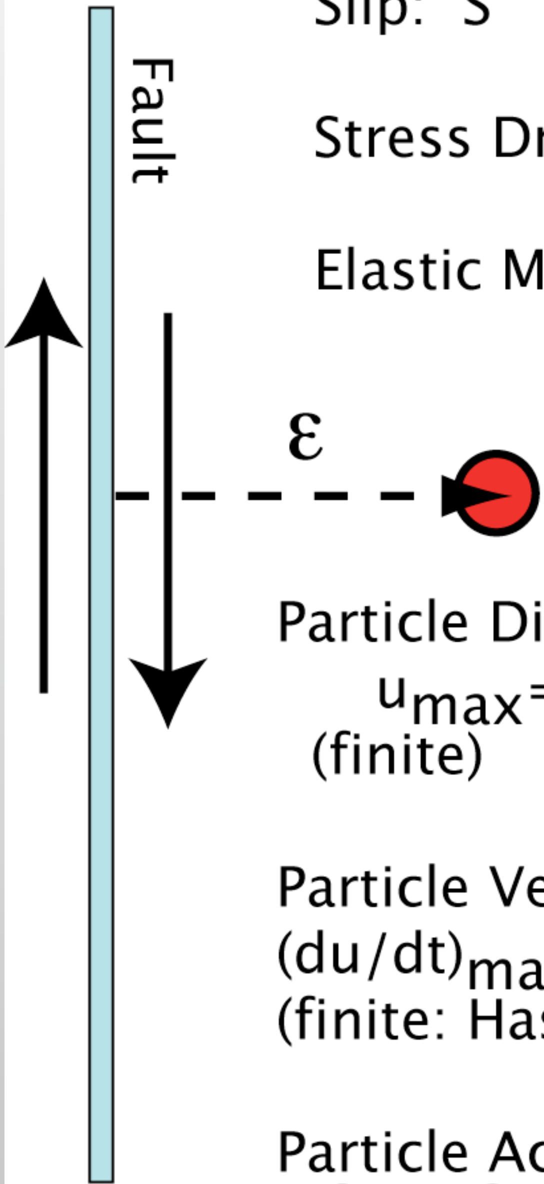
$$\sin 2\ i_\xi = 2 \sin i_\xi \cos i_\xi$$

Haskell & Kostrov Slip Rate

The farfield (which contributes most of the ground motion even in the near-source region) displacement amplitude is proportional to the slip rate on the fault.



Near Fault Ground Motion



Slip: S

Stress Drop: $\Delta\sigma$

Elastic Medium: σ, V_S

Particle Displacement

$u_{max} = \lim_{\varepsilon \to 0} = S/2 \sim (\Delta\sigma/\mu)r$

(finite)

Particle Velocity

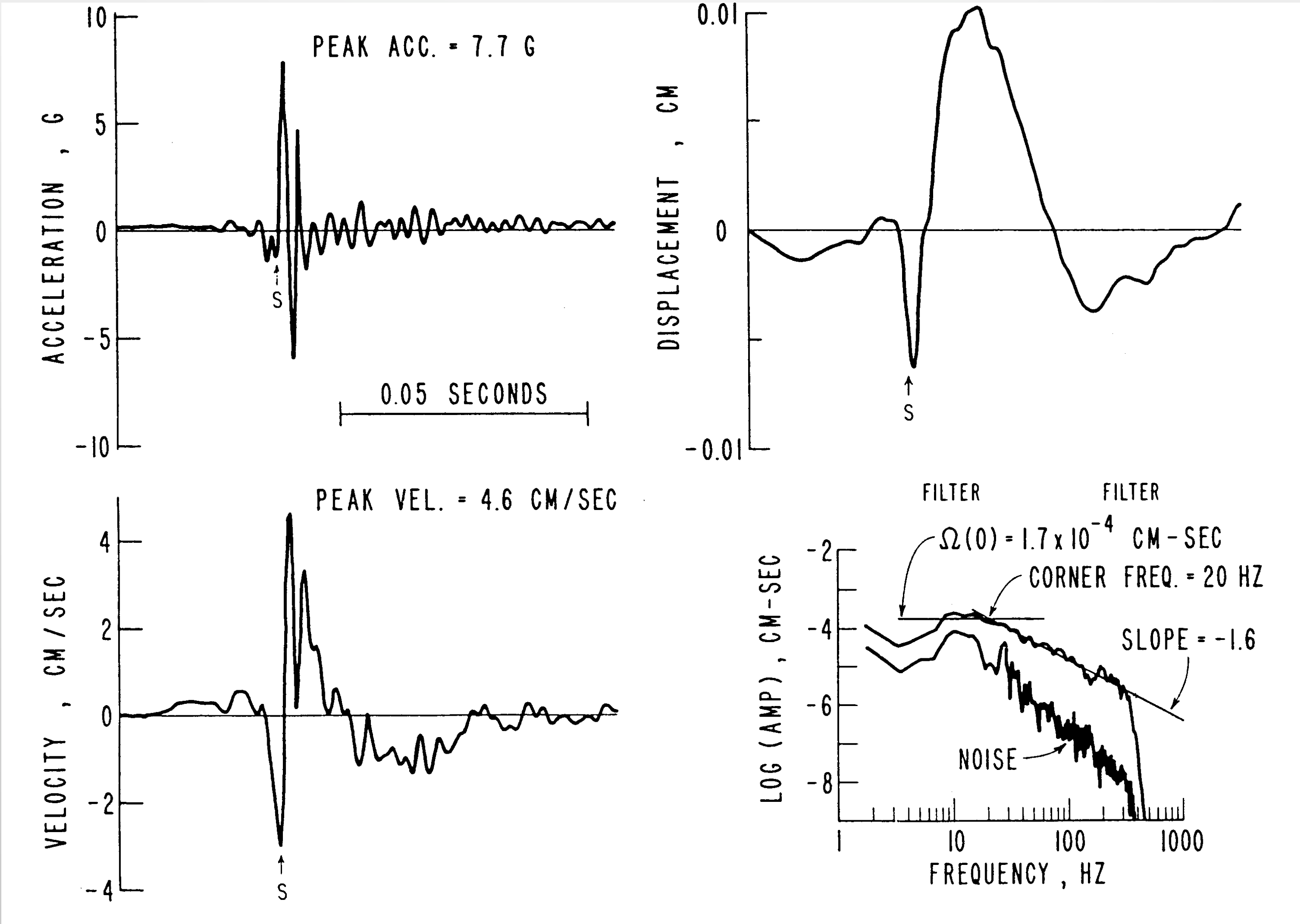
$(du/dt)_{max} = \lim_{\varepsilon \to 0} = (dS/dt)/2 \sim (\Delta\sigma/\mu)V_S$

(finite: Haskell, Brune; infinite: Kostrov)

Particle Acceleration

$(d^2u/dt^2)_{max} = \lim_{\varepsilon \to 0} = ???$

Peak Ground Motions



McGarr, et al., 1981

PGA and PGV Yucca Mountain

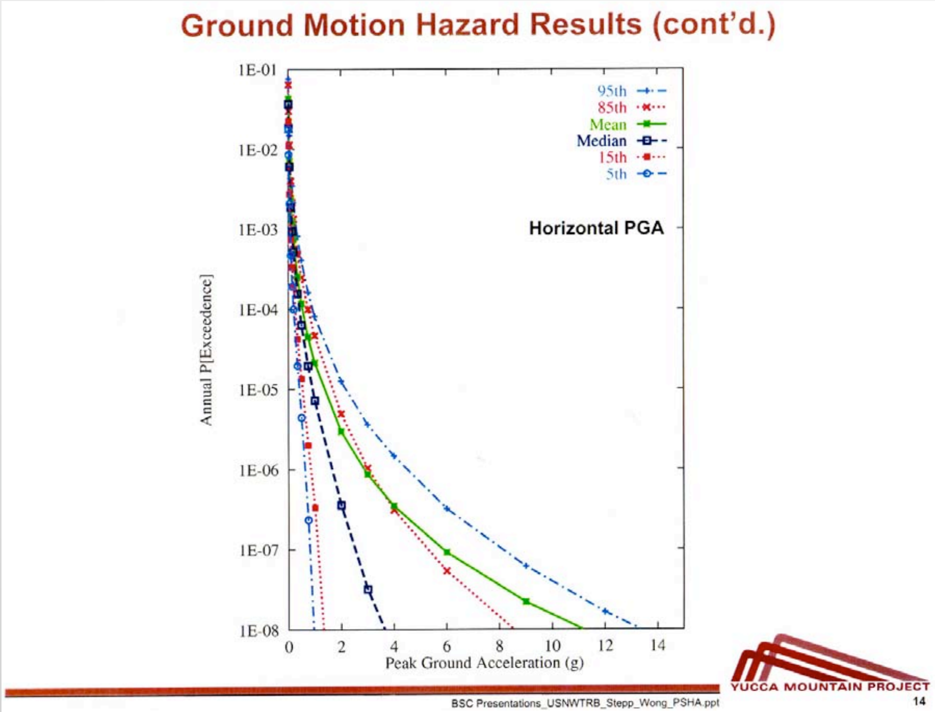


Figure 3. Probability of exceedance calculations for horizontal PGA at Site A, a hypothetical, reference rock outcrop site at the repository horizon (Stepp et al., 2001). From the 1998 Yucca Mountain PSHA extrapolated to 10^{-8}/yr by Stepp and Wong (2003).

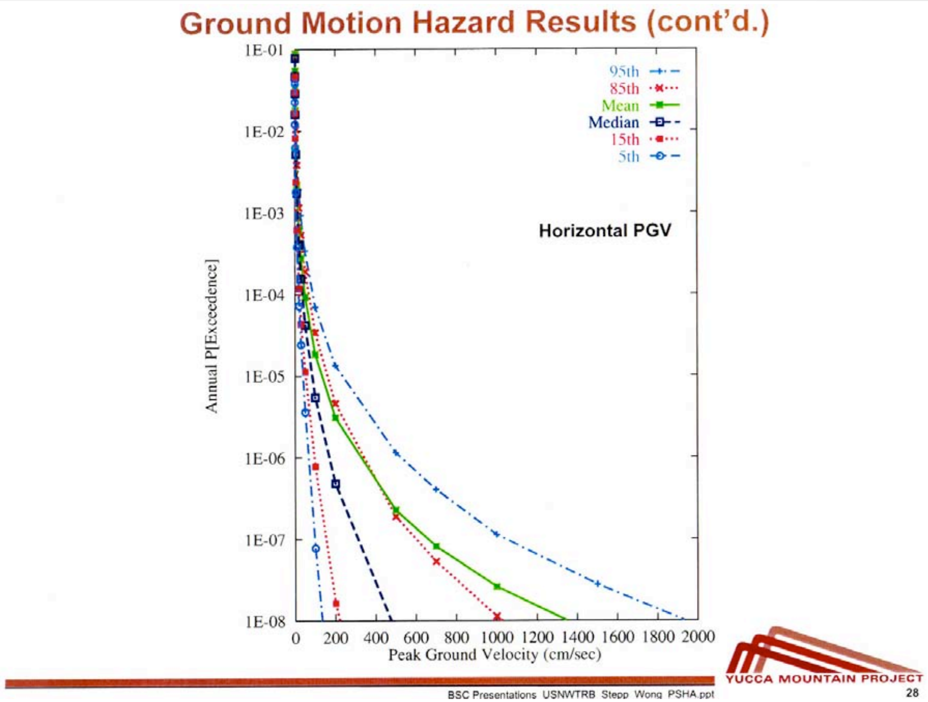
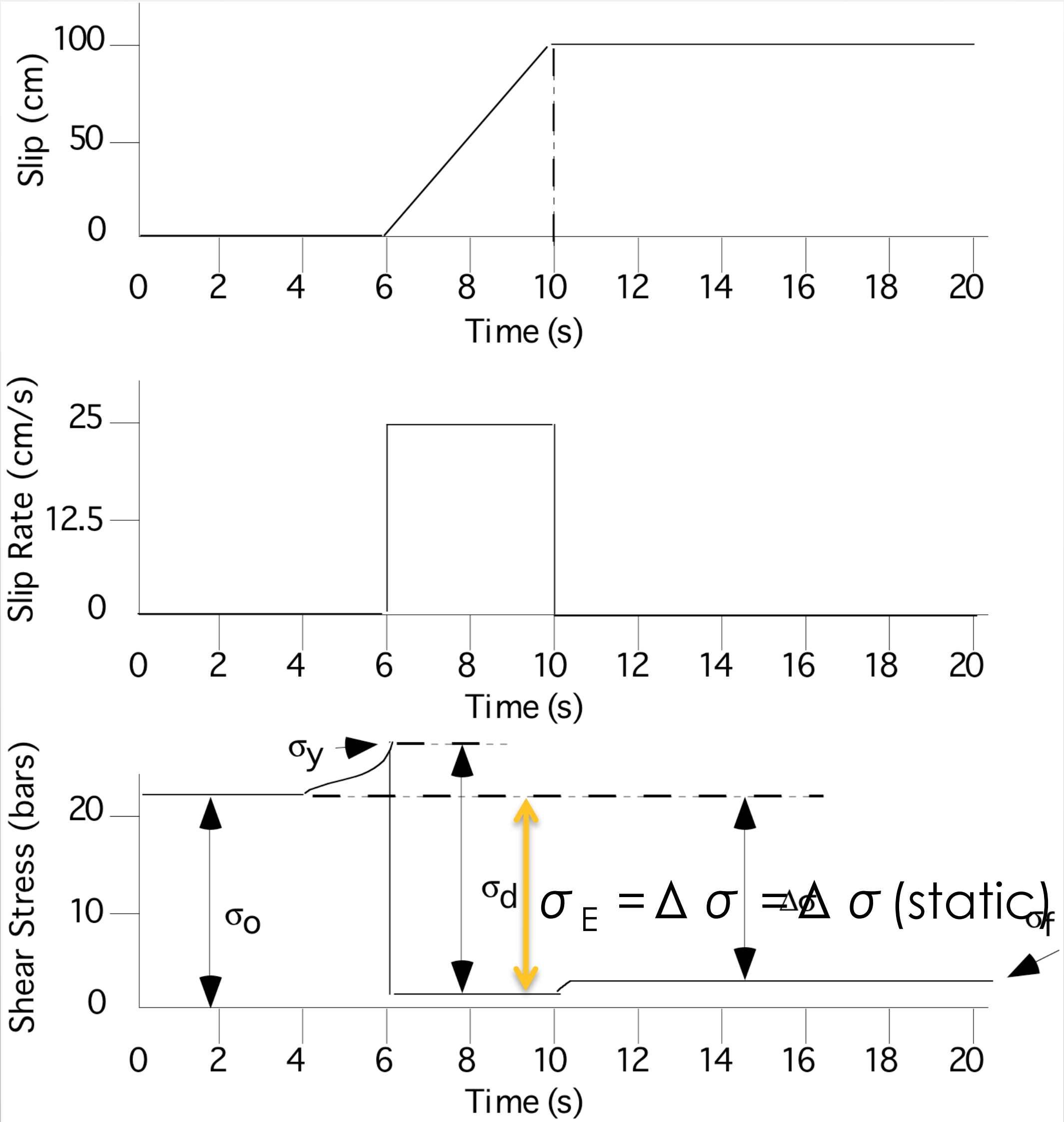


Figure 4. Same as Figure 3, but for horizontal PGV.

Slip and Stress at a Point on the Fault



$\sigma_E = \Delta\sigma = \Delta\sigma\,(\text{static}) = \sigma_0 - \sigma_1$

Stress Drop and Moment

Table 2-1: Stress Drop and Seismic Moment for Three Fault Geometries

	Circular (radius, r)	Strike-Slip	Dip-Slip
$\Delta\sigma$	$\left(\frac{7\pi}{16}\right)\mu\left(\frac{\bar{s}}{r}\right)$	$\left(\frac{2}{\pi}\right)\mu\left(\frac{\bar{s}}{W}\right)$	$\left(\frac{4}{\pi}\right)\left(\frac{\lambda+\mu}{\lambda+2\mu}\right)\mu\left(\frac{\bar{s}}{W}\right)$
M_O	$\left(\frac{16}{7}\right)\Delta\sigma r^3$	$\left(\frac{\pi}{2}\right)\Delta\sigma W^2 L$	$\left(\frac{\pi}{4}\right)\left(\frac{\lambda+2\mu}{\lambda+\mu}\right)\Delta\sigma W^2 L$

Linear Elasticity

Stress and strain are related in an isotropic, linear elastic medium by a single relation (Hooke's Law)

$$\sigma_{ij} = \lambda \delta_{ij} \varepsilon_{kk} + 2\mu \varepsilon_{ij}$$

where λ and μ are Lame's parameter and the shear modulus, respectively.

$$\varepsilon_{\kappa\kappa} = \varepsilon_{11} + \varepsilon_{22} + \varepsilon_{33}$$

$$\delta_{ij} = 1 \quad \text{if} \quad i = j$$

$$= 0 \quad \text{if} \quad i \neq j$$ Kroneker delta

$$\varepsilon_{ij} = \frac{1}{2}\left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i}\right)$$

In a homogeneous medium λ, μ are not functions of position (x_1, x_2, x_3).

For obvious reasons σ_{ij} $(i \neq j)$ are referred to as the shear stresses

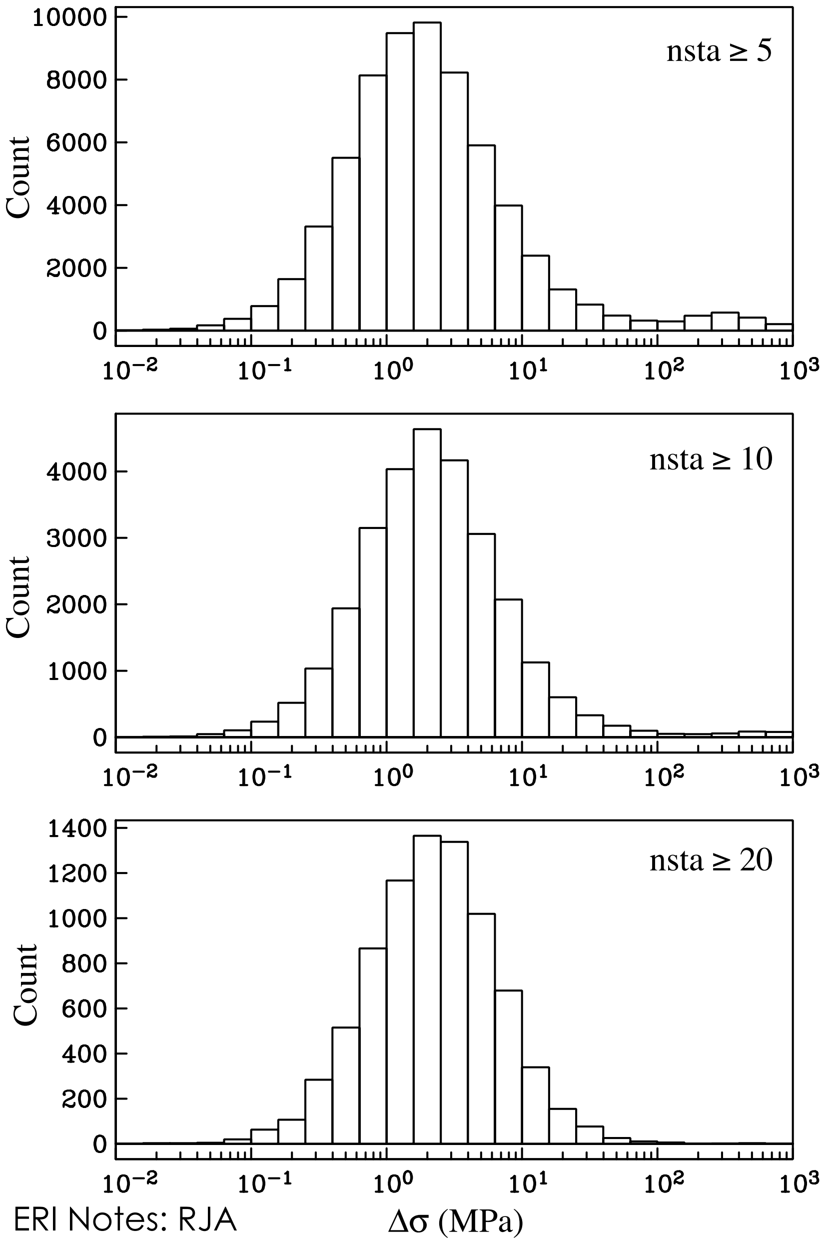
$$\sigma_{ij} = 2\mu \varepsilon_{ij} \quad i \neq j.$$

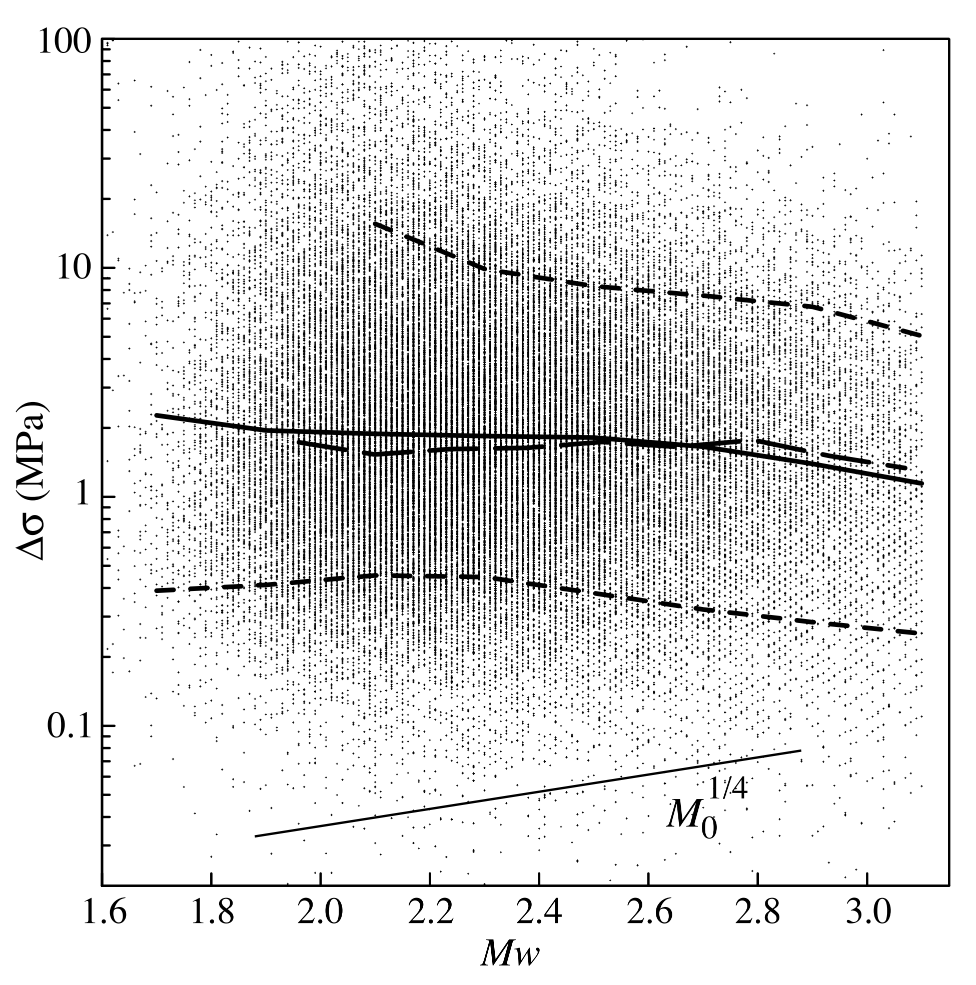
However, if $i = j$

$$\sigma_{ij} = \sigma_{ii} = \lambda \varepsilon_{kk} + 2\mu \varepsilon_{ii}$$

(No summation on i.)

Stress Drops: Southern California





ERI Notes: RJA

Shearer, Prieto, Hauksson, 2007

Global Stress Drops

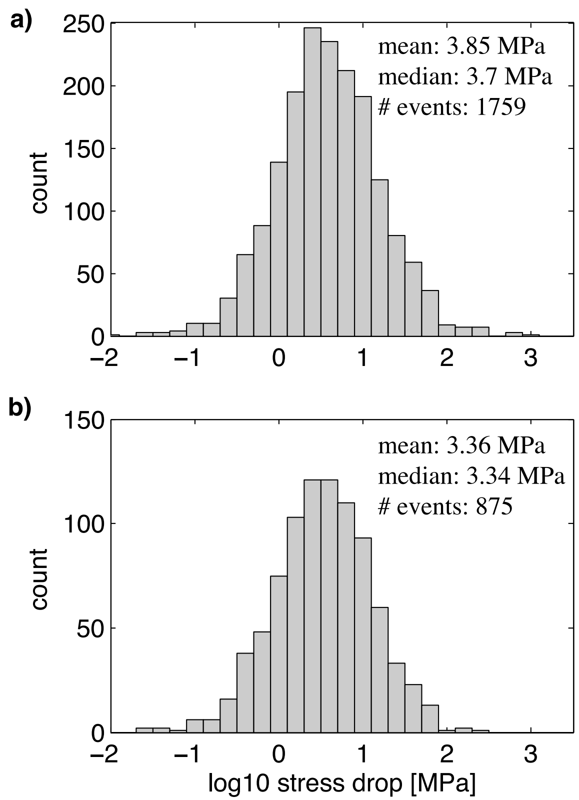


Figure 6. Histogram of logarithmic stress-drop estimates for a different minimum number of required stations. a) For at least three stations per event. b) For at least 20 stations per event.

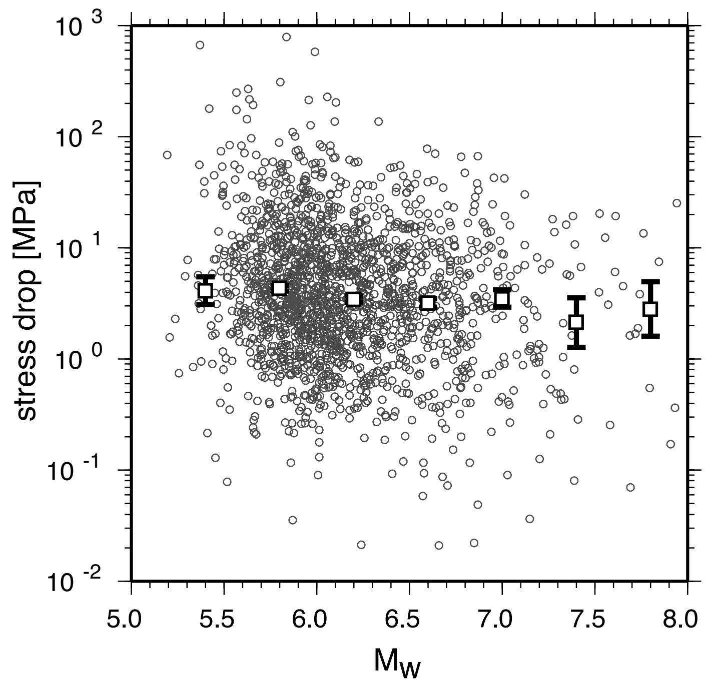
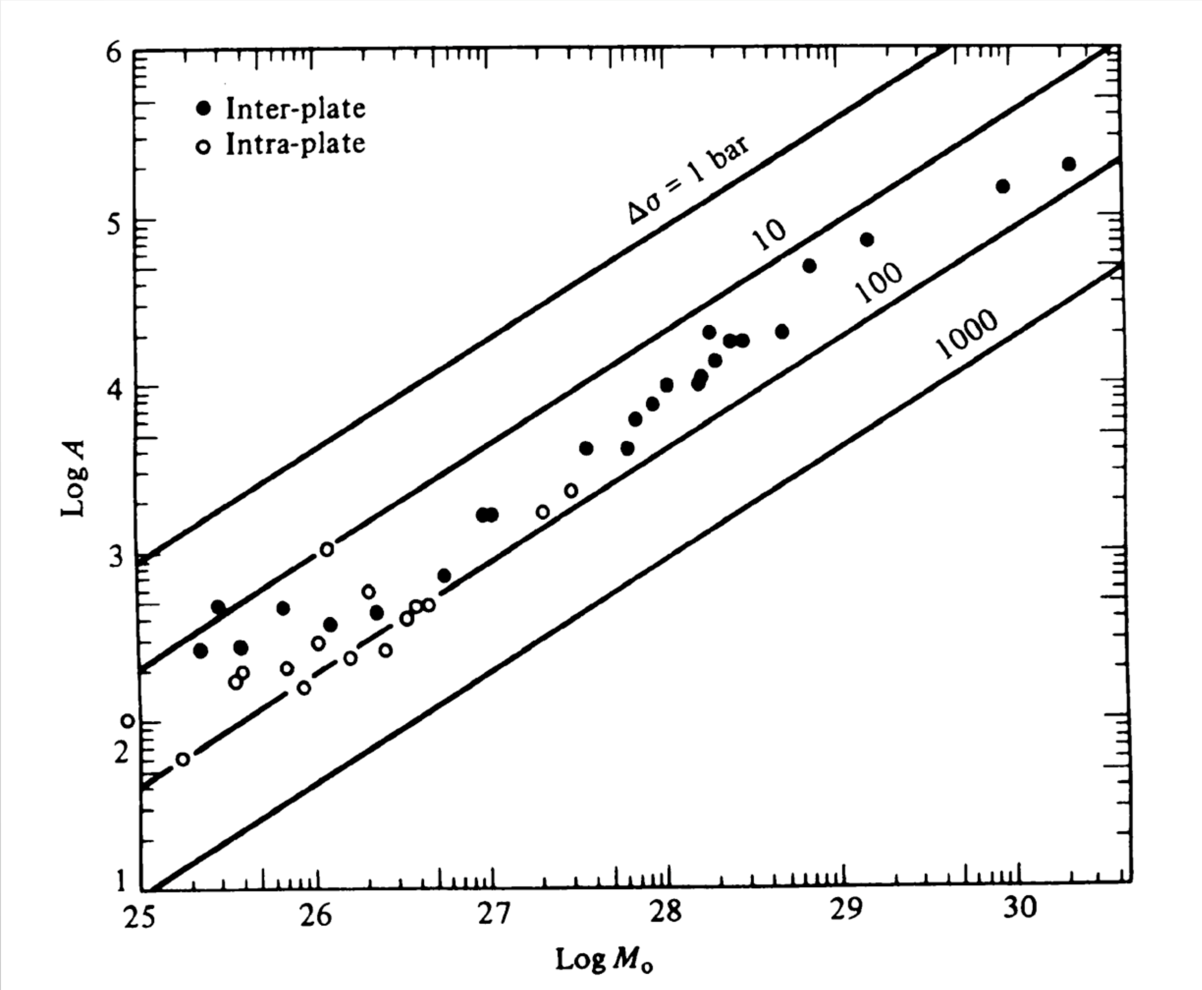


Figure 7. Stress-drop versus moment. The mean of 100 bootstrap-resampled median stress drops for bins of 0.4 in moment magnitude is shown by the white squares. Error bars denote the standard errors from bootstrap resampling. Note the general independence of stress drop and moment over the magnitude range of the data.

Allmann and Shearer, 2009

Stress Drop World Wide Eqks



ERI Notes: RJA

Kanamori and Anderson, 1975

Global Stress Drops

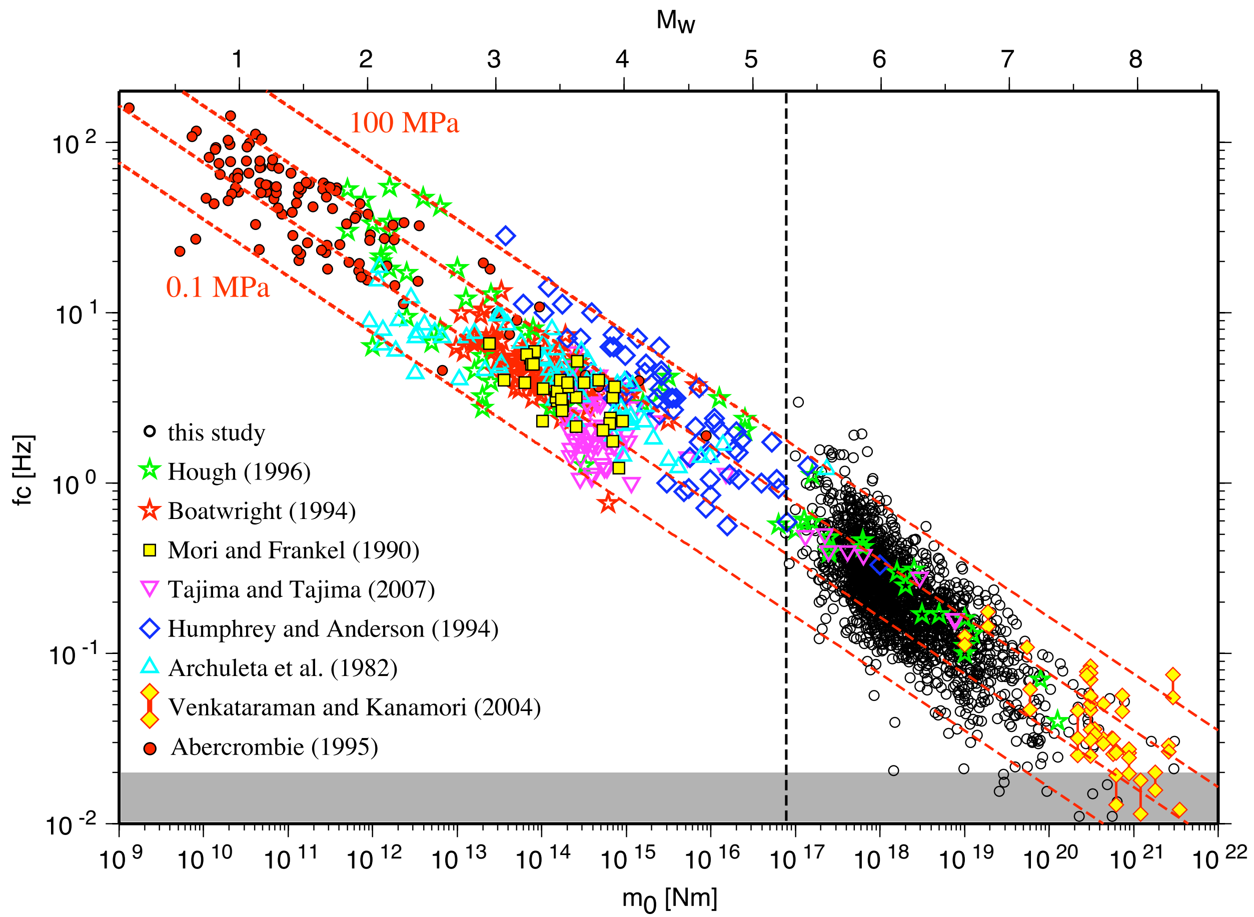


Figure 8. Corner frequency versus seismic moment (lower scale) and moment magnitude (upper scale). The red dashed lines show constant stress drops of 0.1, 1, 10, and 100 MPa. The gray shaded area shows the resolution limit of our data. The vertical dashed line marks the lower magnitude cutoff of our data. The results of this study are plotted as open black circles. All other different shaped and colored symbols show data from various other studies. The data suggest self-similarity over a wide moment range.

Allmann and Shearer, 2009

Finite Fault Parameters

Table 2.2: Average Fault Parameters (from Hasegawa, 1974)

Magnitude M_s	M_o (dyne-cm)	Length (km)	Width (km)	Slip (cm)
5.5	3×10^{24}	6	5	30
6.0	1.4×10^{25}	10	8	53
6.5	5×10^{25}	18	11	75
7.0	2×10^{26}	35	15	120
7.5	4×10^{27}	60	50*	400

[redacted]

Worldwide each year follows the Gutenberg-Richter statistics:

$$N = 10^{8.2 - M_s}$$

where N is the number of earthquakes per year with surface wave magnitude greater than M_S; the b value is 1.0.

Simply apply the above formula to determine the approximate number of earthquakes worldwide for a given magnitude.

For example, M_S 6: $N=10^{8.2 - 6} = 10^{2.2} = 158$.

Magnitude M_S	Number Greater than M_S per year
8.0	2 (1.6 by the formula, but eq'ks are quantized)
7	16
6	160
5	1600
4	16000
3	160000

[redacted]

Brune's Spectral Model

$$\sigma(x,t) = \sigma H(t - x/\beta)$$
$$\sigma(x,t) = \mu \partial u / \partial x$$
$$u = 0 \qquad t<0$$
$$u(t) = (\sigma/\mu)\beta t \qquad t>0$$

The Fourier spectrum of u(t) is

$$\Omega(\omega) = \int_0^\infty (\sigma/\mu)\beta t \exp^{-i\omega t} = -\left(1/\omega^2\right)\left(\sigma\beta/\mu\right)$$

The initial particle velocity is:

$$\dot{u}(t) = \left(\sigma/\mu\right)\beta$$

The inverse Fourier transform of $u(t)$ *over a finite frequency band*

$$u(t) = \left(1/2\pi\right)\left(\sigma\beta/\mu\right)\int_{-\omega_s}^{\omega_s}\left(1/\omega^2\right)\exp(i\omega t)\,d\omega$$

$$\ddot{u}(t) = \left(1/2\pi\right)\left(\sigma\beta/\mu\right)\int_{-\omega_s}^{\omega_s}\exp(i\omega t)\,d\omega$$

$$\ddot{u}(t) = \left(1/\pi\right)\left(\sigma\beta/\mu\right)\omega_s\left(\sin\omega_s t/\omega_s t\right)$$

For $\omega_s = 10\,Hz$ *and* $\sigma = 10\,MPa$, $\ddot{u}(f=10) \simeq 2g$

Brune's Spectral Model

$$u(x=0,t)=(\sigma/\mu)\beta\tau(1-e^{-t/\tau})$$

$$\dot{u}(x=0,t)=(\sigma/\mu)\beta e^{-t/\tau}$$

The Fourier transform of displacement is

$$\Omega(\omega)=(\sigma/\mu)\beta\omega^{-1}(\omega^{2}+\tau^{-2})^{-1/2}$$

where $\tau=O(r/\beta)$

The farfield displacement is proportional to slip rate and the effect of diffraction from the finite size (r) of the fault is approximated by multiplying by $e^{-\alpha t}$.

Of course we now have to consider the reduced time $t''=t-R/\beta$ *and we will multiply by a factor* $f\bullet(r/R)$

$$u(R,t)=f\bullet(r/R)(\sigma/\mu)\beta t'' e^{-\alpha t''}$$

$$\dot{u}(R,t)=f\bullet(r/R)(\sigma/\mu)\beta \quad at\ t''=0$$

The Fourier transform of the displacement u(R,t) is

$$\Omega(\omega)=f\bullet(r/R)(\sigma\beta/\mu)(\omega^{2}+\alpha^{2})^{-1}$$

f and α *are determined by requiring the long-period limit of the spectral density agrees with that from a double-couple determined from a dislocation.*

$$\Omega_{s}(\omega)=\frac{R_{\theta\vartheta}M_{0}}{(4\pi\rho\beta^{3}R)}$$

Brune's Spectral Model

The average spectrum is given by:

$$\langle \Omega_s(\omega) \rangle = \langle R_{\theta\vartheta} \rangle (\sigma\beta / \mu)(r / R)(\omega^2 + (2.34\beta / r)^2)^{-1}$$

The corner frequency $f_c = (1/2\pi)(2.34\beta / r) = 0.37\beta / r$

At zero frequency, the spectrum must be the same as that from the double-couple dislocation.

$$\langle \Omega_s(0) \rangle = \langle R_{\theta\vartheta} \rangle M_0 \Big/ \left(4\pi\rho\beta^3 R\right)$$

or

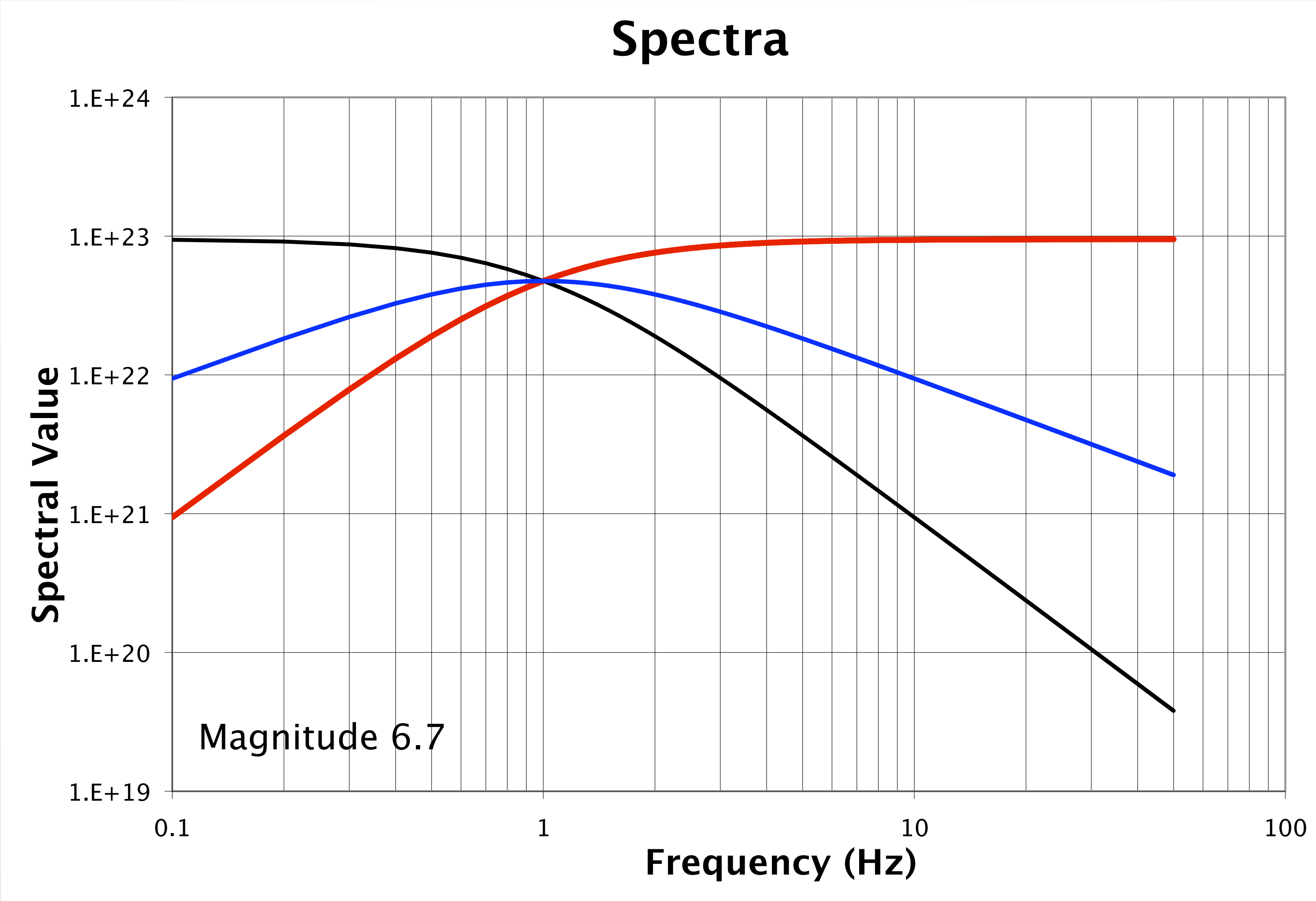
$$M_0 = \left(4\pi\rho\beta^3 R\right) \langle \Omega_s(0) \rangle \Big/ \langle R_{\theta\vartheta} \rangle$$

Brune : $f_c = 0.37\beta / r$

Madariaga(1976) $f_c = 0.21\beta / r$

Madariaga(1979) $f_c = 0.28\beta / r$

Brune Spectrum



Computing Corner Frequency and Seismic Moment from Spectrum

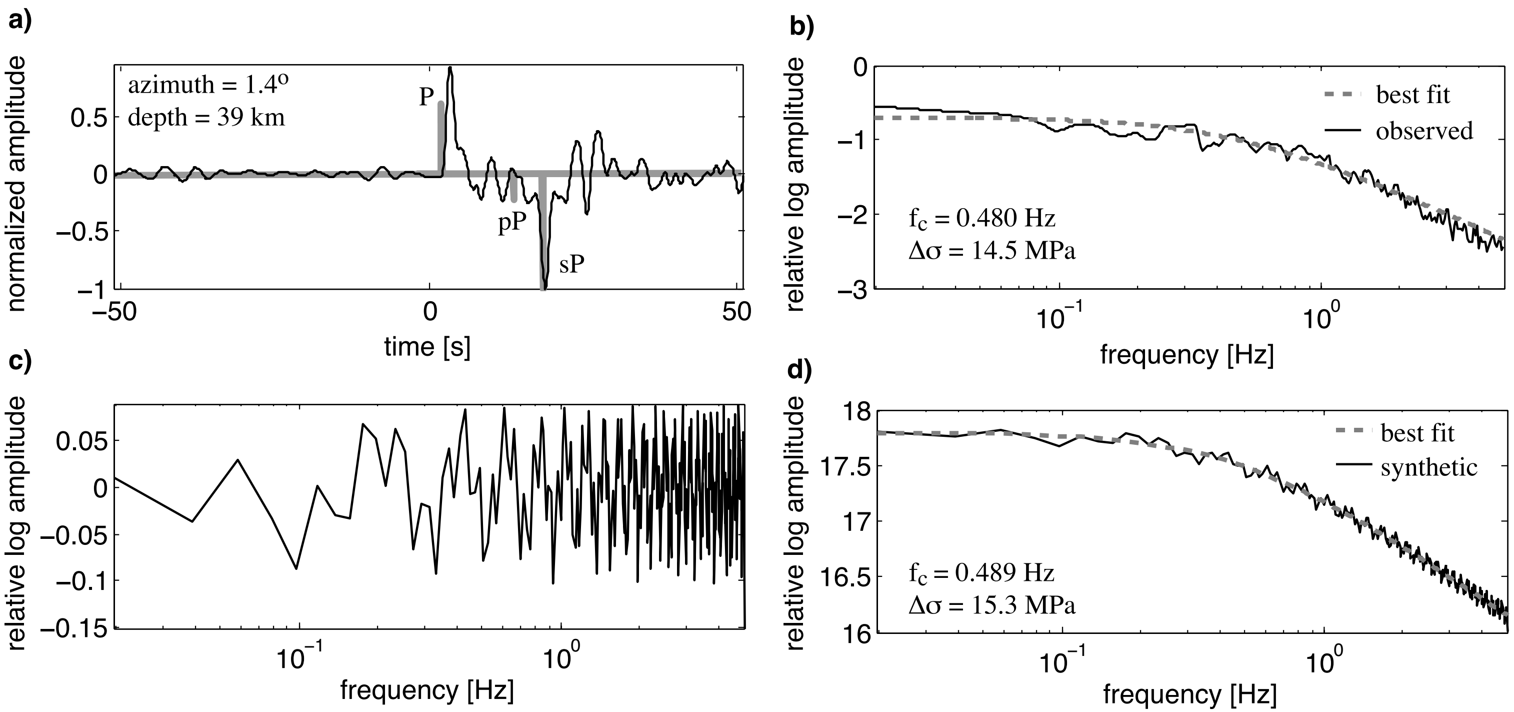
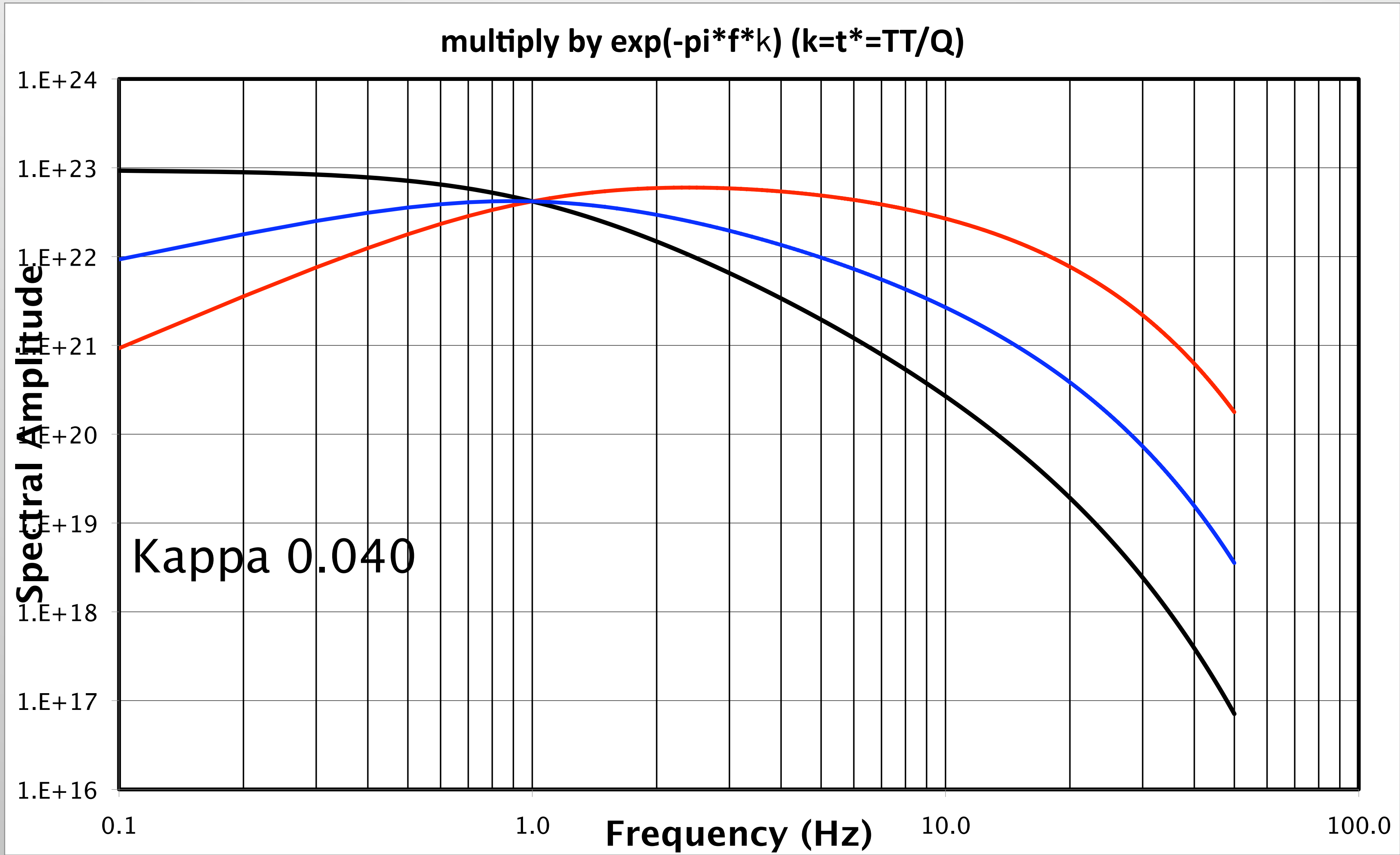


Figure 5. a) Windowed normalized time series of one example trace with strong depth phases (black). The computed stick seismogram is shown in bold grey. b) Computed EGF-corrected source spectra for the example trace (black). The best-fitting theoretical model (dashed grey) has a corner frequency of 0.48 Hz and a stress drop of 14.5 MPa. c) Demeaned spectrum of the stick seismogram. d) Sum of best-fitting theoretical model and spectrum of stick seismogram (black). The dashed grey line shows the new best-fitting model. Note the small difference in f_c compared to b).

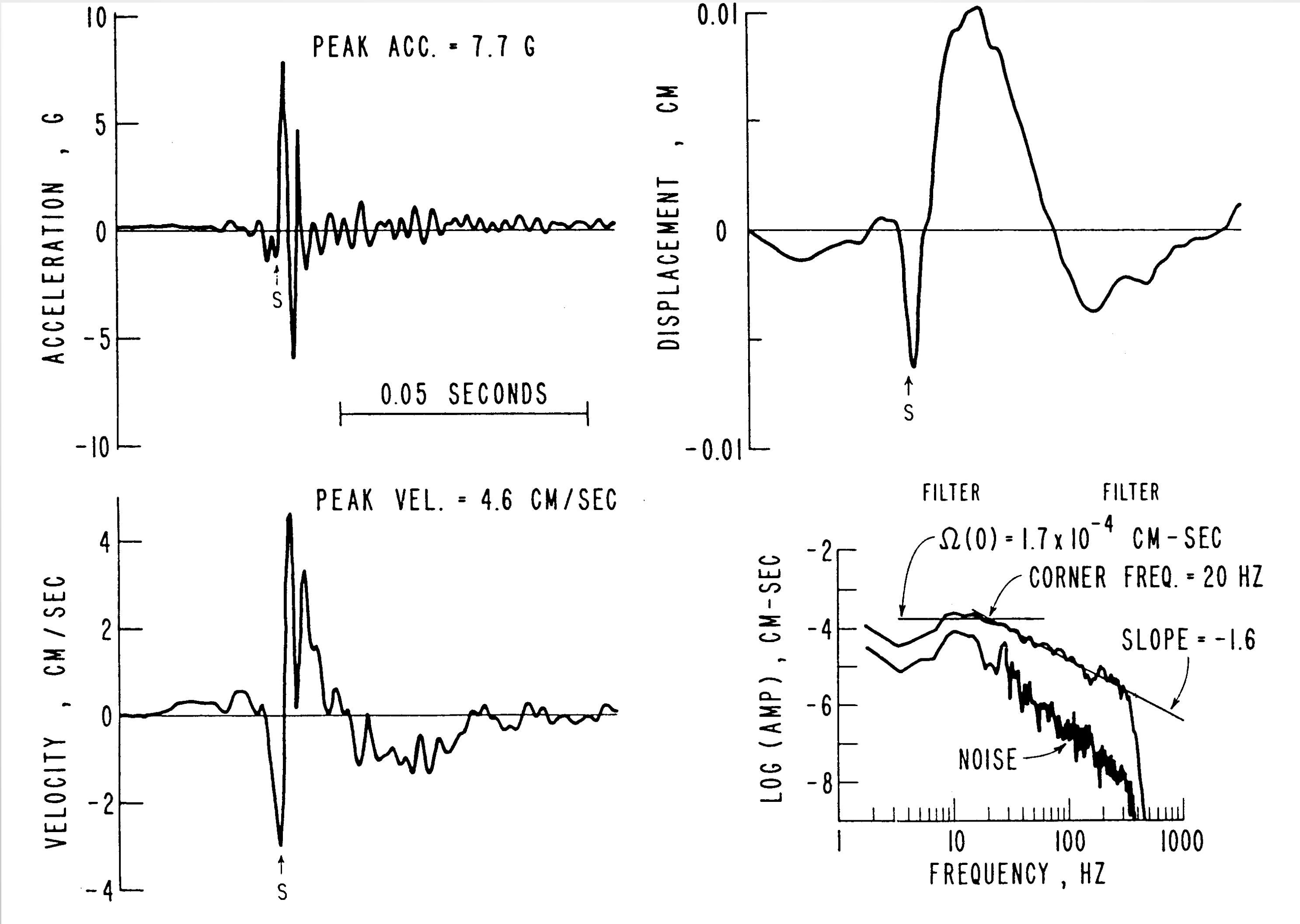
Allmann and Shearer, 2009

Brune Spectrum with Kappa

$\kappa = t^* =$ travel time/Q



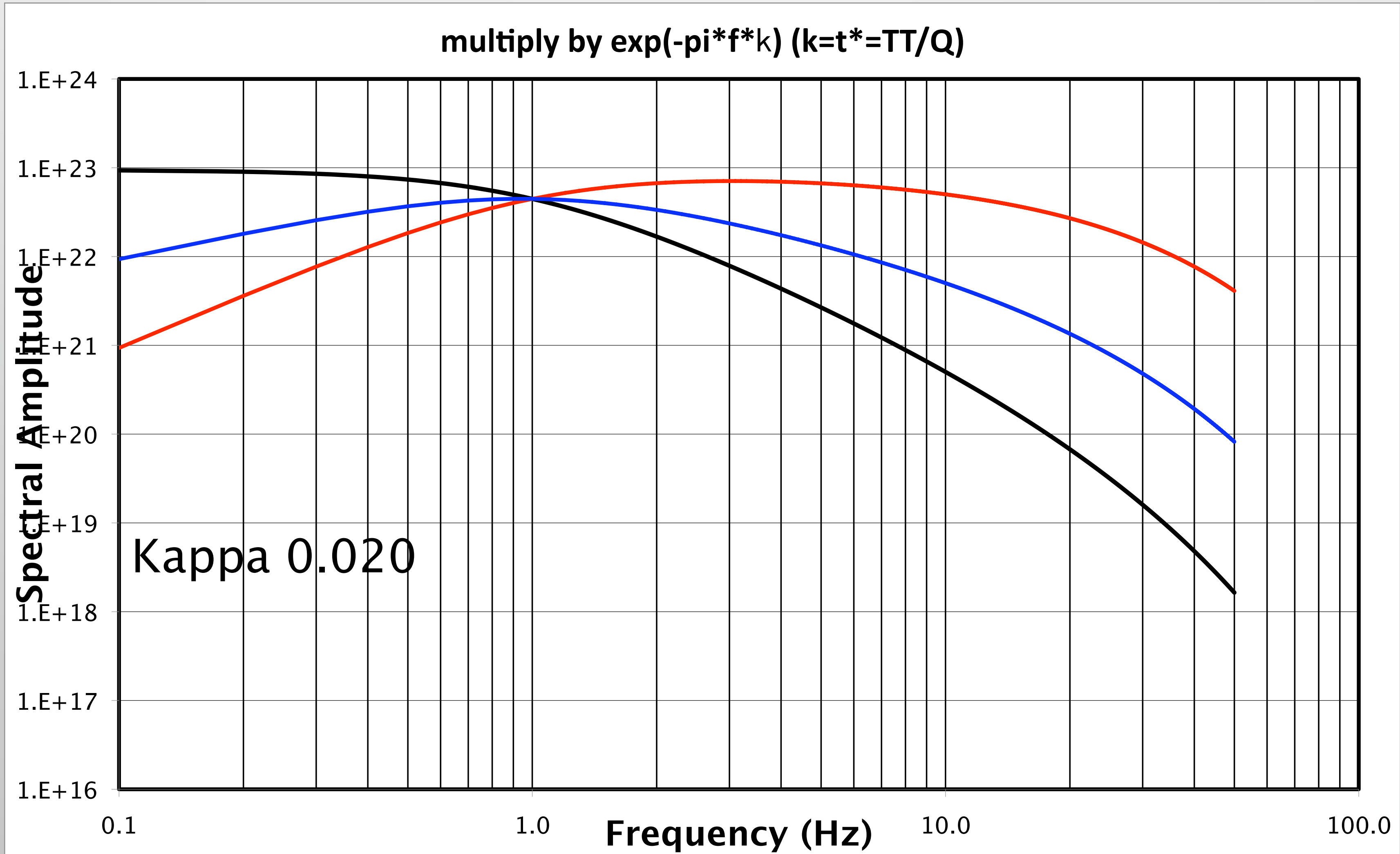
Acceleration: Gold Mine



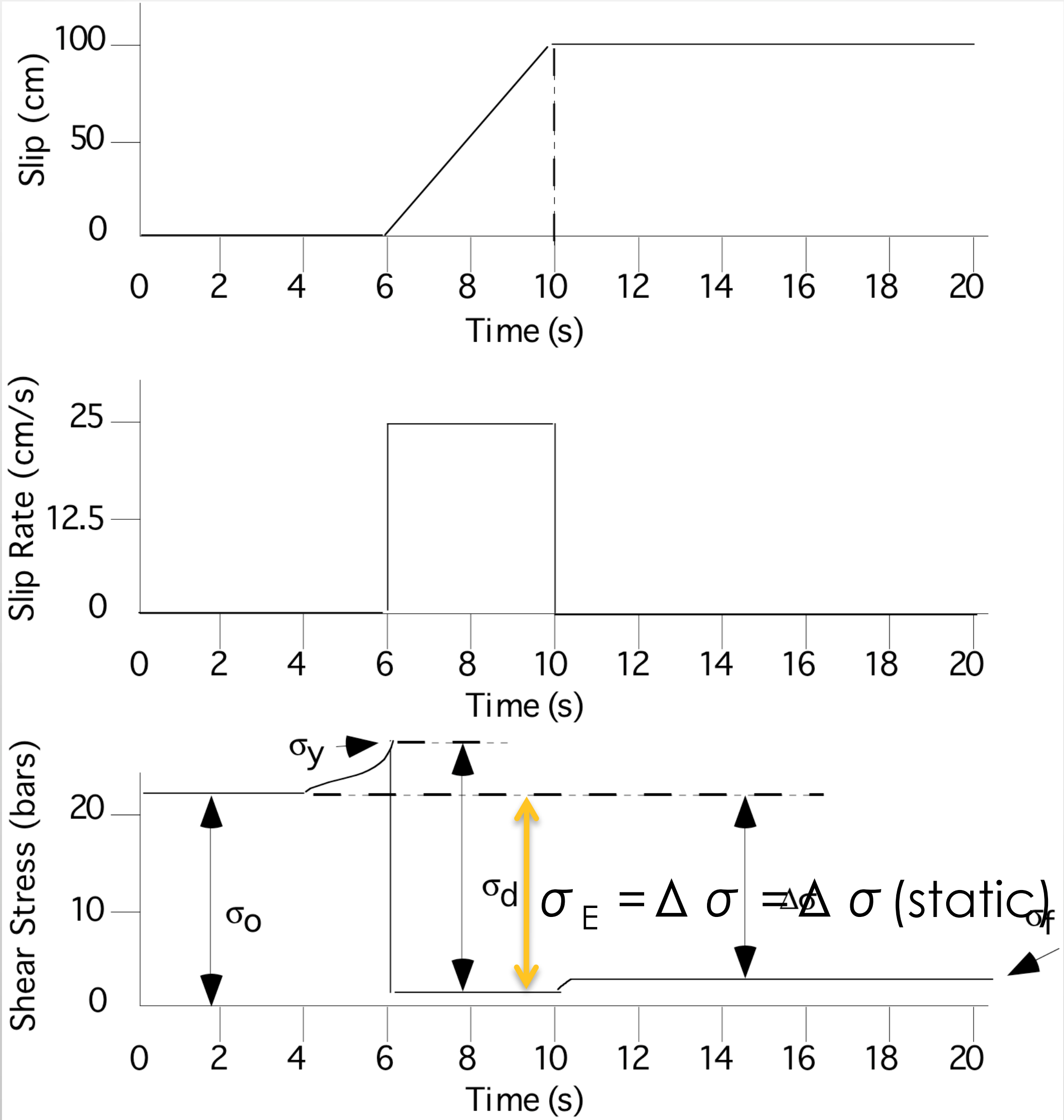
McGarr et al., 1981

Brune Spectrum with Kappa

$\kappa = t^* =$ travel time/Q



Slip and Stress at a Point on the Fault



$\sigma_E = \Delta\sigma = \Delta\sigma\,(\text{static}) = \sigma_0 - \sigma_1$

Brune's Spectral Model

$$M_0 = \left(16/7\right)\sigma r^3$$
$$f_c = 0.37\beta / r$$
$$M_0 = \left(16/7\right)\sigma (0.37\beta / f_c)^3$$
$$\sigma = \left[(7/16)(1/0.37\beta)^3\right] M_0 f_c^3$$
$$Brune: f_c = 0.37\beta / r$$
$$Madariaga(1976) f_c = 0.21\beta / r$$
$$Madariaga(1979) f_c = 0.28\beta / r$$

Thus the difference in stress drop between Brune and Madariaga is $(0.37/0.21)^3 = 5.5$

Stress and Energy (1)

$$\sigma_a = \mu \frac{E_s}{M_0}$$: Apparent Stress

$$\sigma_a \le \Delta\sigma / 2$$

$$E_s = \tfrac{1}{2}(\sigma_0 + \sigma_1)DA - \sigma_f DA$$: Elastic energy - Frictional Energy

Orowan's model: $\sigma_f = \sigma_1$

$$E_s = \tfrac{1}{2}(\sigma_0 - \sigma_1)DA$$: Only the stress difference is measured

$$E_s = \tfrac{1}{2}(\Delta\sigma)DA$$

$$DA = M_0 / \mu$$

$$\tfrac{1}{2}(\Delta\sigma) = \mu \frac{E_s}{M_0}$$

Stress and Energy (2)

Following Randall, BSSA, 1973: The spectral theory of seismic sources

$$\Omega_S(0) = \langle R_{\theta\vartheta} \rangle M_0 / (4\pi\rho\beta^3 R) = \langle R_{\theta\vartheta} \rangle M_0 / (4\pi\mu\beta R)$$

$$|\Omega_S(\omega)| = \left| \int_{-\infty}^{\infty} u(t) e^{-i\omega t} dt \right| \le \int_{-\infty}^{\infty} |u(t)| e^{-i\omega t} dt = \Omega_S(0) \textit{ provided that}$$

the displacement pulse does not change sign.

Following Randal (1972)

$$\textit{let } x{=}\omega/2\pi f_c \quad \Omega_S(\omega) = \left[\langle R_{\theta\vartheta} \rangle M_0 / (4\pi\rho\beta^3 R) \right] F(x)$$

$$\textit{where } F(x){=}(1{+}x^2)^{-1} \quad \textit{for Brune's Model}$$

$$\textit{Note: for P waves } \Omega_P(\omega) = \left[\langle R_{\theta\vartheta} \rangle M_0 / (4\pi\rho\alpha^3 R) \right] F(x)$$

$$\textit{and } f_c = 0.37\alpha / r$$

$$E_R(PorS) = (1/2)(\langle R_{\theta\vartheta}^2 \rangle M_0^2 f_c^3) / (\rho c^5) \int_0^{\infty} x^2 F^2(x) \textit{ where } c = \alpha \textit{ or } \beta$$

$$E_R(S) \big/ E_R(P) = 3\alpha^2 \big/ 2\beta^2$$

Stress and Energy (3)

Thus the total radiated energy is

$$E_R = (11/9)(4\pi/5)(\langle R^2_{\theta\vartheta}\rangle M_0^2 f_c^3)/(\rho\beta^5)\int_0^\infty x^2 F^2(x)$$

This is valid for any choice of spectral model.

Assuming the stress drops from σ^0 *to* σ^1: $\sigma = \sigma^0 - \sigma^1$

The energy available for seismic radiation from a circular fault

$$W = (1/2)(\sigma M_0/\mu)$$

Using $M_0 = (7/16)\sigma r^3 \Rightarrow \sigma = (16/7)(M_0/r^3)$

Substitute σ *this into the expresssion for W*

$$W = \left(7\pi^3 M_0^2 f_c^3\right)/4\rho\beta^5 k^3 \quad \textit{where } 2\pi f_c = k\beta/r$$

$$W = E_R$$

$$\left(7\pi^3 M_0^2 f_c^3\right)/4\rho\beta^5 k^3 = (11/9)(4\pi/5)(\langle R^2_{\theta\vartheta}\rangle M_0^2 f_c^3)/(\rho\beta^5)\int_0^\infty x^2 F^2(x)$$

which leads to an expression for k

$$k \doteq 17.3 \Big/ \int_0^\infty x^2 F^2(x)$$

Stress and Energy (4)

For Brune's spectral model one finds

$k \doteq 2.80$ *as opposed to what Brune derived 2.34 (correction, 1971)*

$f_c = 0.44\beta / 4$ *which is 20% larger.*

Many different models including simple ω^{-2} *(k=2.35) and*

even simple ω^{-3} *models (k=2.96) or spherical models (*ω^{-2}*) with values*

of k~2.7 allow one to estimate E_R

$$E_R \doteq 3.0\left(M_0^2 f_c^3 \Big/ \rho\beta^5 \right)$$

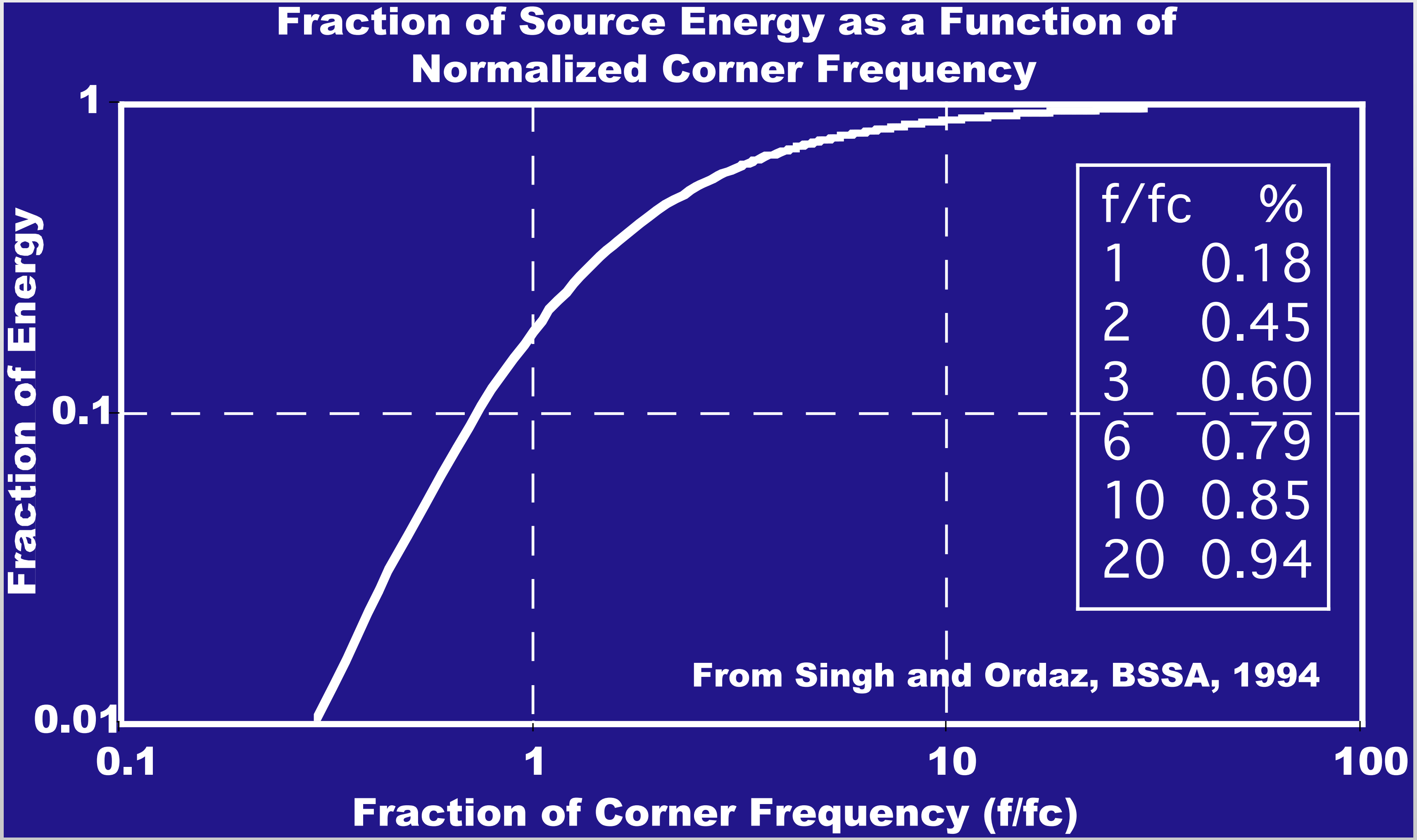
Recall that we started with an energy for a circular fault

$$W = (1/2)(\sigma M_0 / \mu) = E_R = 3.0\left(M_0^2 f_c^3 \Big/ \rho\beta^5 \right)$$

$$\sigma = 2\mu\left(E_R \Big/ M_0 \right)$$ *twice the "apparent stress" (Wyss and Molnar, 1972)*

$$\sigma \doteq 6.0\left(M_0 f_c^3 \Big/ \beta^3 \right)$$

Fraction of Energy in Source Spectrum



Representation Theorem

$$u_n(\boldsymbol{x},t) = \int_{-\infty}^{\infty} d\tau \iint_{\Sigma} [u_i(\xi,\tau)]\, c_{ijpq} \nu_j \partial G_{np}(\boldsymbol{x},t-\tau;\xi,0)/\partial \xi_q \, d\Sigma$$

Aki and Richards, (3.2)

$$\hat{\mathbf{a}} \bullet \ddot{\mathbf{u}}^s = \ddot{f}_r * \int_{\mathbf{y}(t,\mathbf{x})} \left[c^2 \left(\frac{d\mathbf{s}_r}{dq} \bullet \mathbf{G}_a^s \right) + c^2 \left(\frac{d\mathbf{G}_a^s}{dq} \bullet \mathbf{s}_r \right) + \frac{dc}{dt} \left(\mathbf{s}_r \bullet \mathbf{G}_a^s \right) \right] dl$$

Slip Rate Time Function

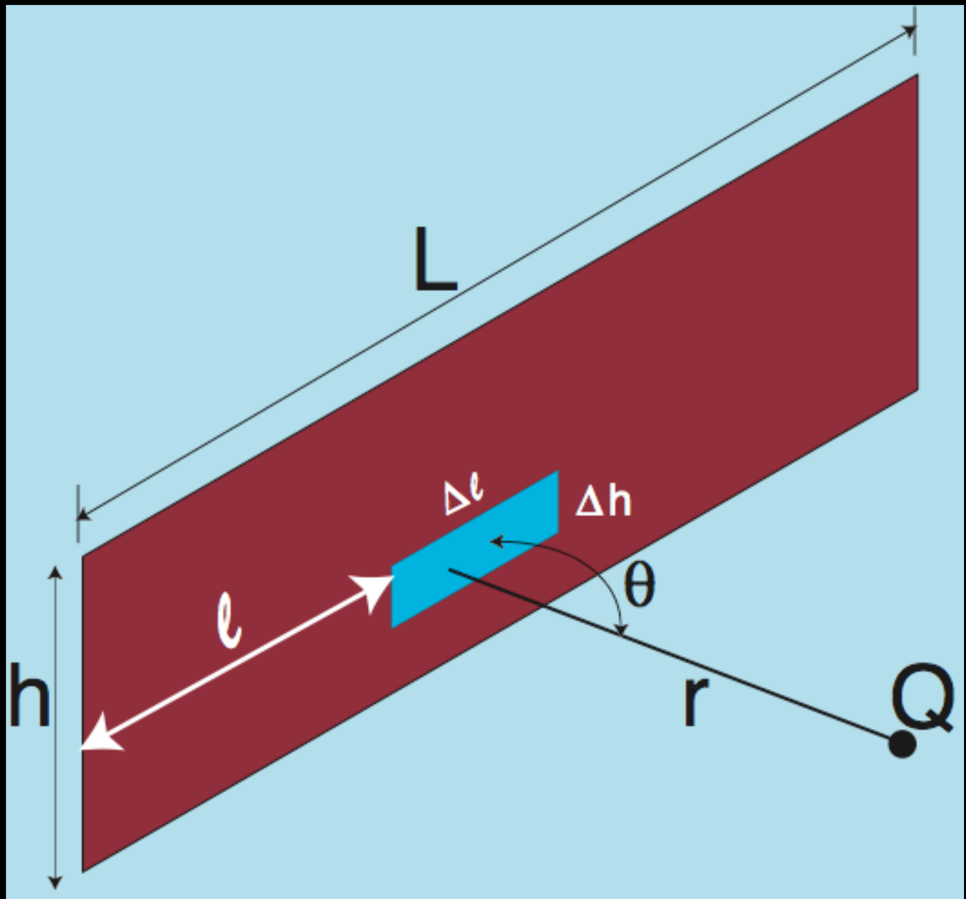
Stress Drop (spatial derivative of slip)

Change in isochrone Velocity--acceleration of the the rupture front

Spudich and Frazer, BSSA, 1984

"Seismic Displacements Near a Fault" Keiiti Aki (J. Geophys. Res., 1968

$$U^l(Q,\omega) = D\Delta h \Delta l \left[A(\omega)\frac{\sin X_a}{X_a}\exp(-i\omega r/a - iX_a - i\omega l/v) + B(\omega)\frac{\sin X_b}{X_b}\exp(-i\omega r/b - iX_b - i\omega l/v) \right]$$



$$X_c = \omega(\Delta l/2)\left[(1/v) - (\cos\theta/c)\right]$$

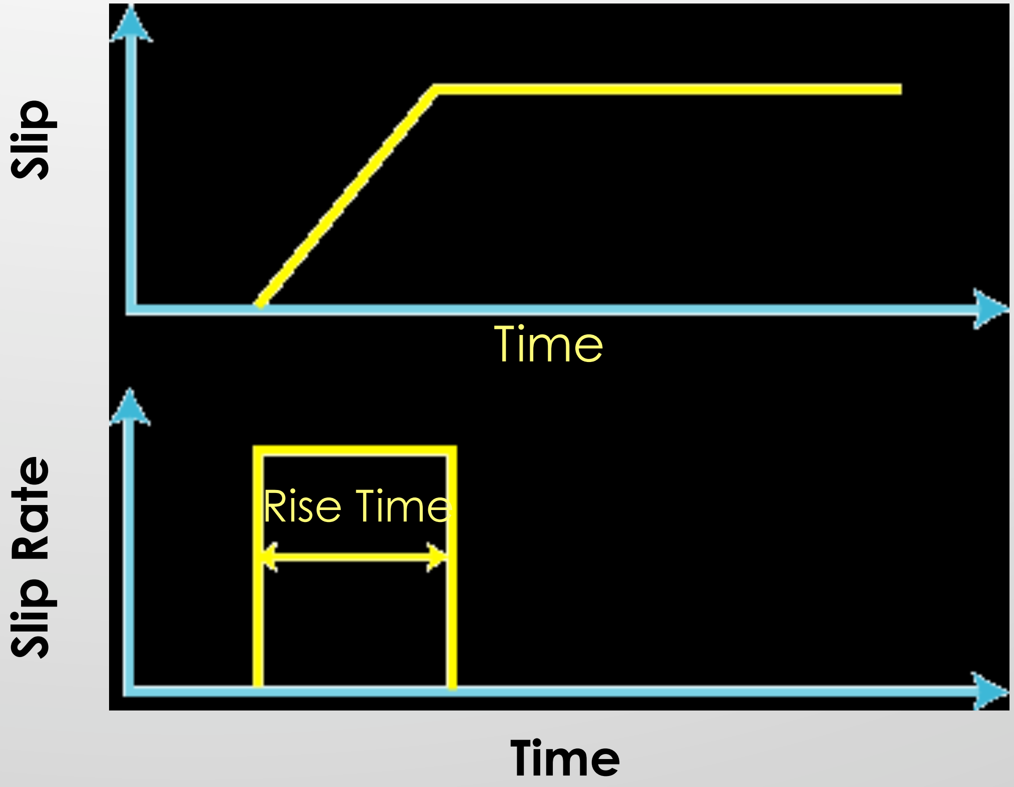
$$D = D_0 \frac{\left(h^2 - Z^2\right)^{1/2}}{h} \qquad 0 < Z < h$$

Total Motion at Q

$$U(Q,t) = FFT^{-1}\left[\sum_l U^l(Q,\omega)\right]$$

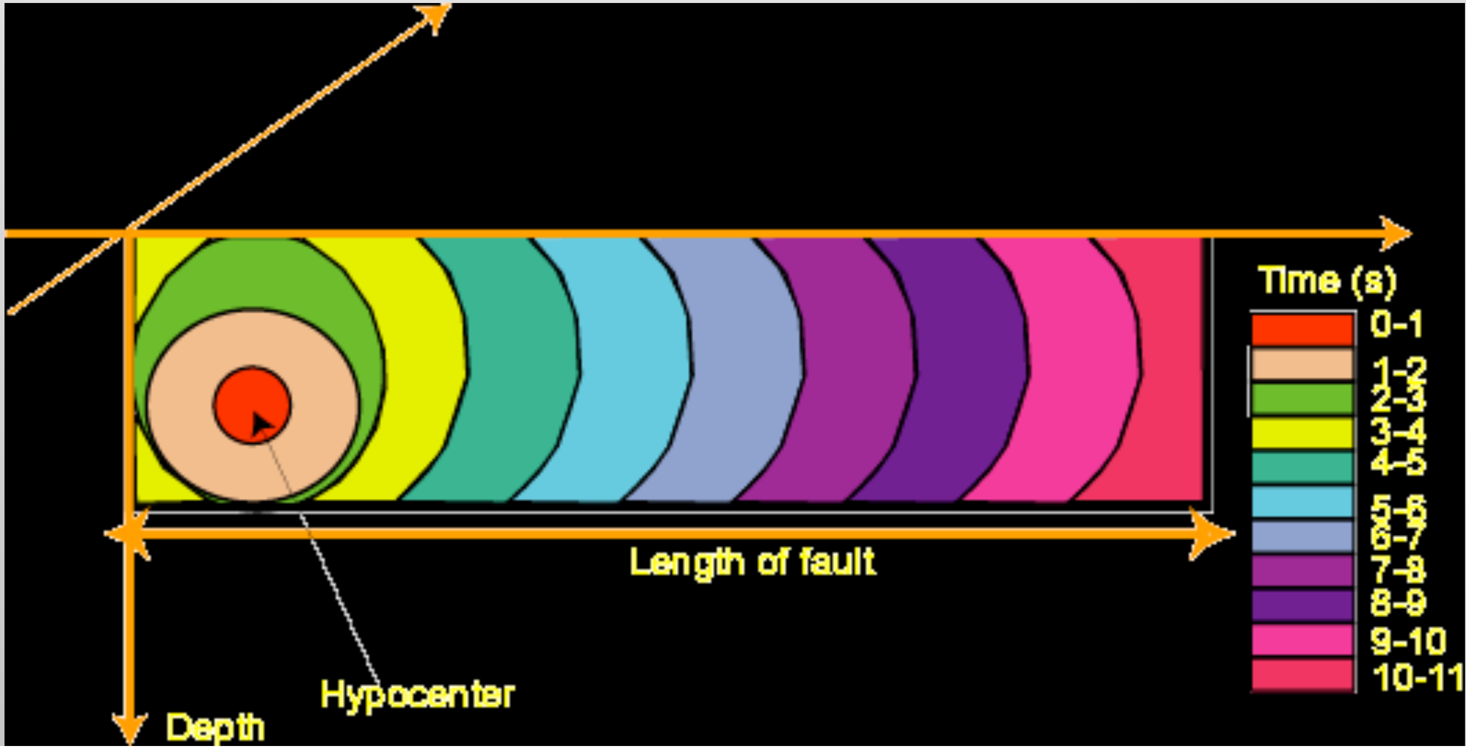
Finite Fault—Kinematic (Haskell, 1964)

- **Fault Geometry: Length, Width**
- **Slip**
- **Rise Time**
- **Rupture Velocity**

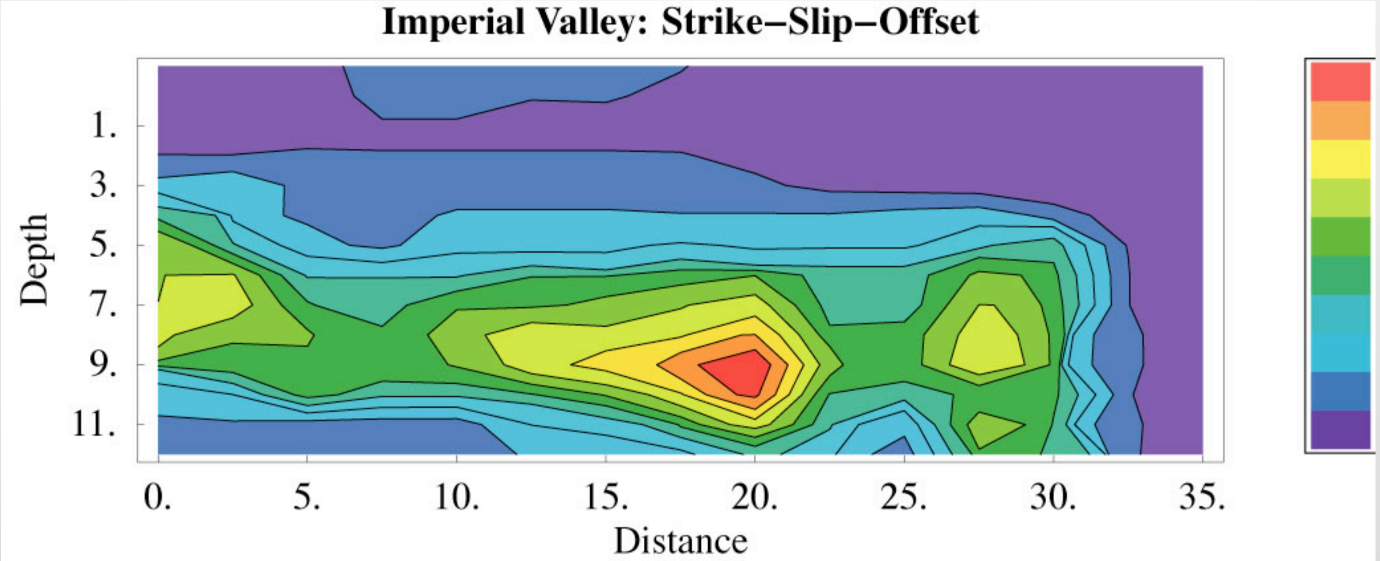


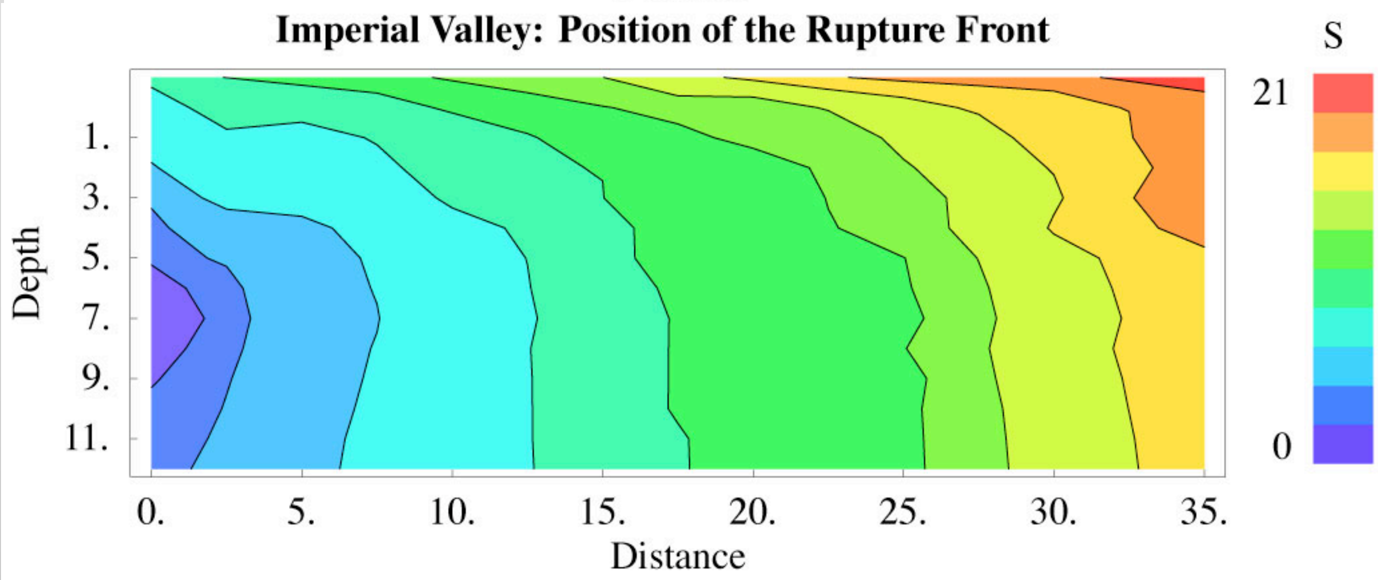
Haskell Model

Haskell Model: A rupture front radiates out from the hypocenter at constant speed. Each point on the fault has the same slip and the same rise time. The time at which a point starts to slip is determined by the distance of that point from the hypocenter divided by the rupture speed.



Imperial Valley Slip and Rupture





 From Archuleta, 1984, JGR

Kinematic Slip Functions

1) Haskell's (1969) ramp

$$\vec{a}(\vec{\xi},t) = \begin{cases} 0 & t - \frac{\xi_1}{v} < 0 \\ \frac{D_o}{T_R} t & 0 < t - \frac{\xi_1}{v} < T_R \\ D_o & T_R < t - \frac{\xi_1}{v} \end{cases} \quad (1.4.4)$$

2) Exponential shape (Brune, 1970)

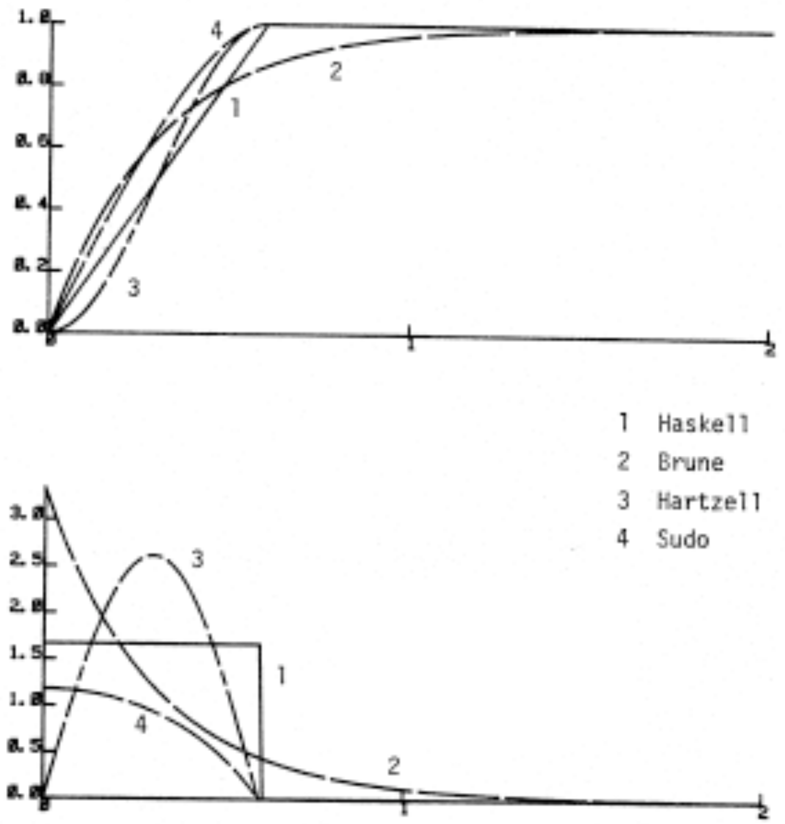
$$\vec{a}(\vec{\xi},t) = \begin{cases} 0 & t - \frac{\xi_1}{v} < 0 \\ D_o(1-e^{-t/T_R}) & 0 < t - \frac{\xi_1}{v} < T_R \\ D_o & T_R < t - \frac{\xi_1}{v} \end{cases} \quad (1.4.5)$$

3) Elliptical (Hartzell, 1978)

$$\vec{a}(\vec{\xi},t) = \begin{cases} 0 & t - \frac{\xi_1}{v} < 0 \\ D_o\sqrt{1 - [(\frac{t}{T_R})^2-1]^2} & 0 < t - \frac{\xi_1}{v} < T_R \\ D_o & T_R < t - \frac{\xi_1}{v} \end{cases} \quad (1.4.6)$$

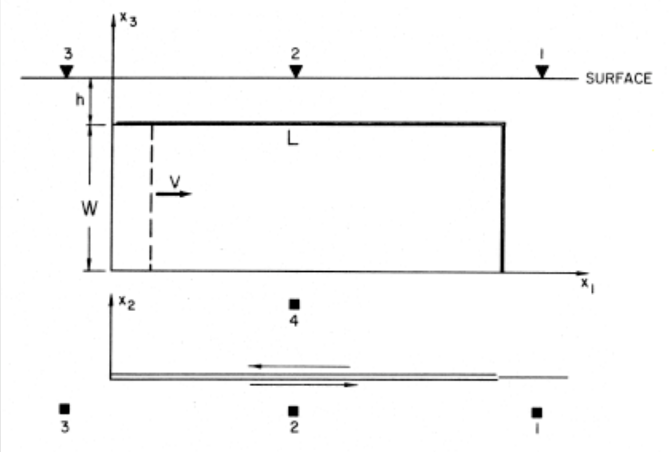
4) Sudo's model (K. Sudo, 1972)

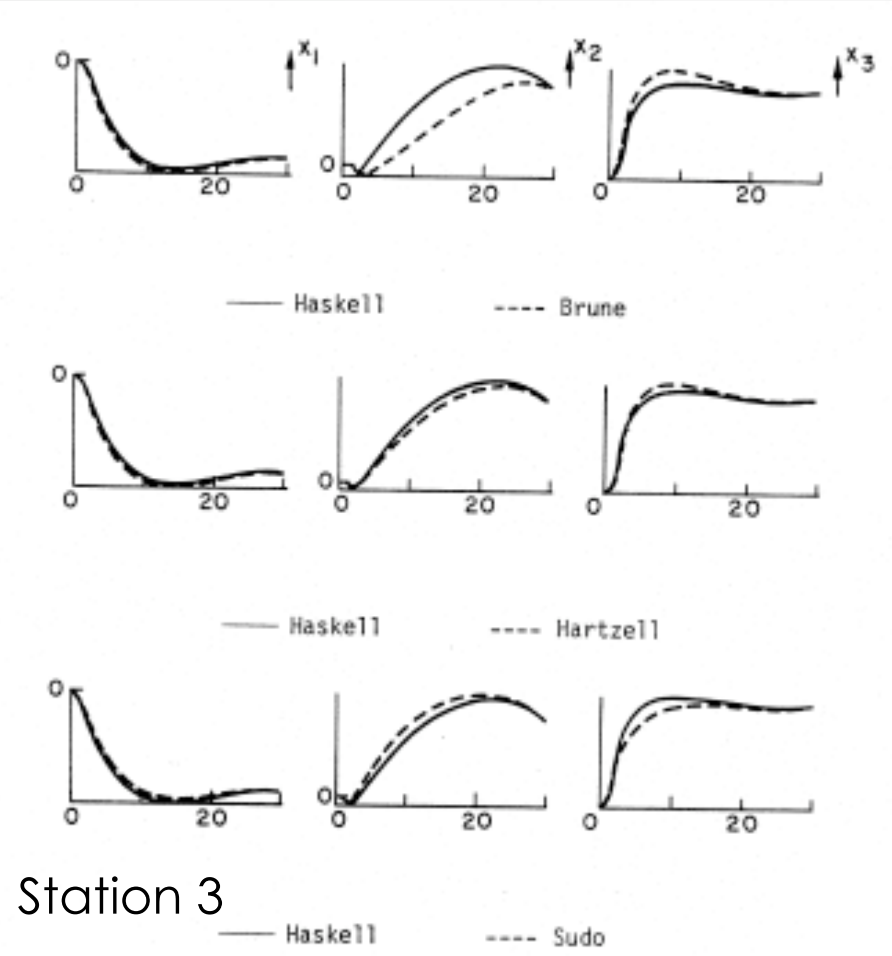
$$\vec{a}(\vec{\xi},t) = \begin{cases} 0 & t - \frac{\xi_1}{v} < 0 \\ \frac{D_o}{2}(1-\cos\pi \frac{t}{T_R}) & 0 < t - \frac{\xi_1}{v} < T_R \\ D_o & T_R < t - \frac{\xi_1}{v} \end{cases} \quad (1.4.7)$$

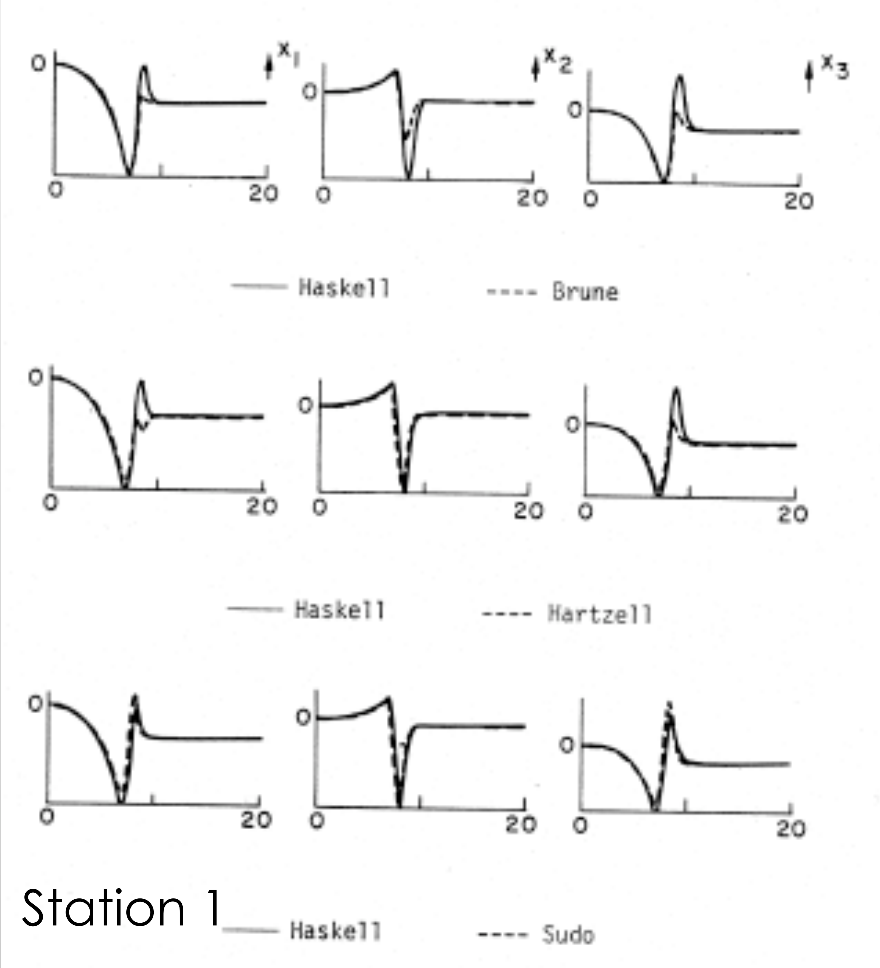


Jordanovski, 1986

Displacements from Kinematic Ruptures with Different Slip Functions

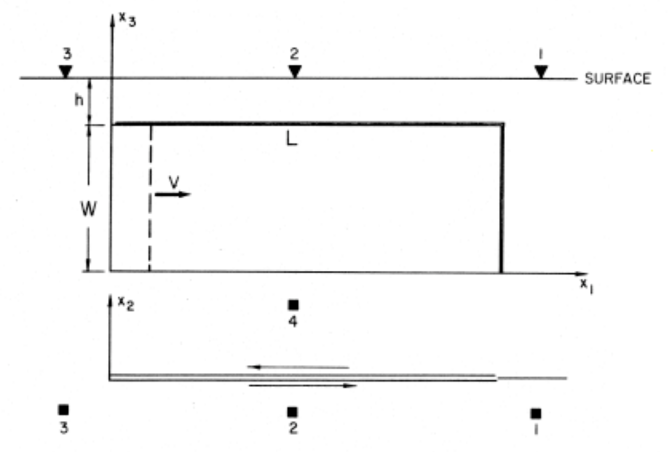


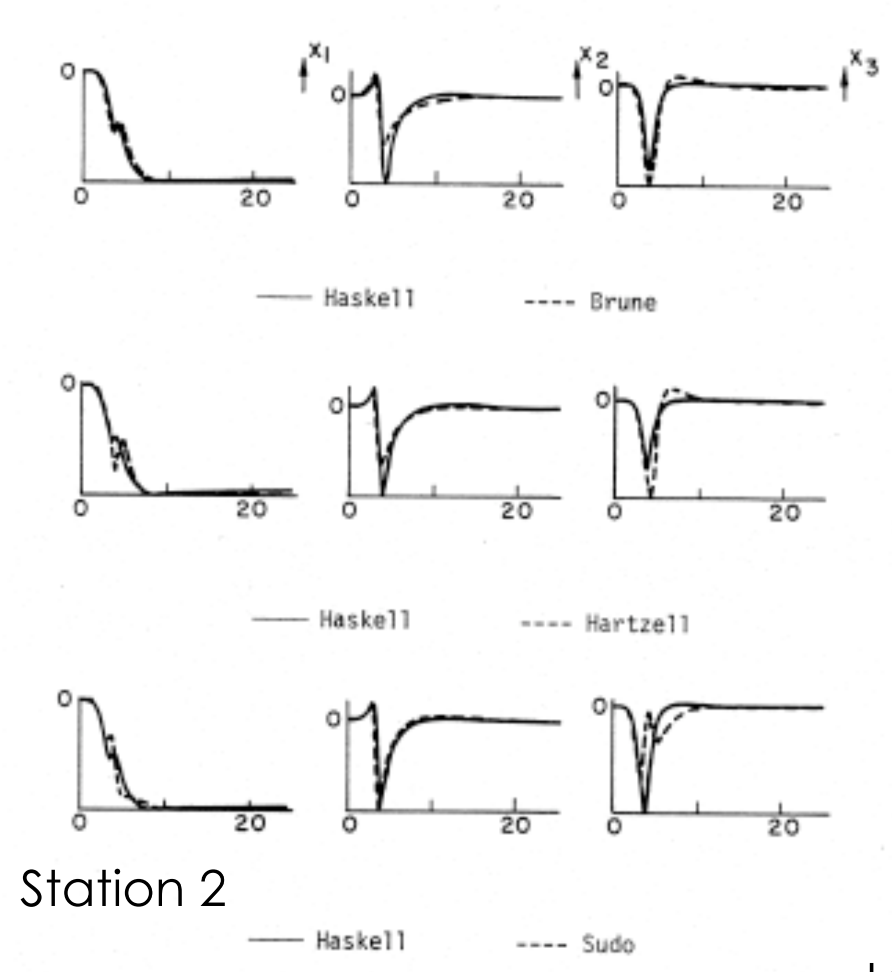




Jordanovski, 1986

Displacements from Kinematic Ruptures with Different Slip Functions





Jordanovski, 1986