Rupture Velocity (3)

$$U(x_i,\omega) = Wi\omega\Delta U(\omega)e^{-i\omega r_0/\beta}\int_0^L \exp\left[-i\frac{\omega\xi}{\beta}\left(\frac{\beta}{v}-\cos\theta\right)\right]d\xi$$

$$let\ \gamma = -\frac{\omega}{\beta}\left(\frac{\beta}{v}-\cos\theta\right)$$

$$\int_0^L \exp\left[-i\frac{\omega\xi}{\beta}\left(\frac{\beta}{v}-\cos\theta\right)\right]d\xi = \int_0^L e^{i\gamma\xi}\,d\xi = \frac{2}{\gamma}\sin\left(\frac{\gamma L}{2}\right)\exp\left(i\frac{\gamma L}{2}\right)$$

$$\frac{2}{\gamma}\sin\left(\frac{\gamma L}{2}\right)\exp\left(i\frac{\gamma L}{2}\right) = L\left(\frac{\sin X}{X}\right)e^{iX}\ \ where\ X = \frac{\gamma L}{2} = -\frac{\omega L}{2\beta}\left(\frac{\beta}{v}-\cos\theta\right)$$

$$U(x_i,\omega) = WL\omega\Delta U(\omega)\frac{\sin X}{X}\exp\left[-i\left(\frac{\omega r_0}{\beta}-X-\frac{\pi}{2}\right)\right]$$

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Rupture Velocity (4)

$$U(x_i,\omega) = WL\omega\Delta U(\omega)\frac{\sin X}{X}\exp\left[-i\left(\frac{\omega r_0}{\beta} - X - \frac{\pi}{2}\right)\right]$$

Note that $\frac{\sin X}{X}$ *is the sinc function* $X = -\frac{\omega L}{2\beta}\left(\frac{\beta}{v} - \cos\theta\right)$ *thus the rupture contributes a* $\frac{1}{\omega}$ *to the decay of the spectrum.*

Suppose $\Delta u(t) = \Delta u H(t)$, *then* $\Delta U(\omega) = \Delta u / i\omega$

For Haskell's model: $\Delta u(t) = \Delta u t / \tau$ *where* τ *is the rise time of the ramp*

$\Delta U(\omega) = \Delta u(1 - e^{-i\omega t}) / \omega^2\tau$: *What is the spectral shape?*

For Brune's dislocation $\Delta u(t) = \Delta u H(t)(1 - e^{-t/\tau})$

$\Delta U(\omega) = \Delta u / (1 + i\omega\tau) i\omega$: *What is the spectral shape?*

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